Model Solutions

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number	Candidate Number

Tuesday 15 January 2019

Morning (Time: 2 hours) | Paper Reference **4MA1/2HR**

Mathematics A

Level 1/2
Paper 2HR
Higher Tier

You must have:
Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- **Calculators may be used.**
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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1/1/1/

P 5 9 0 2 4 A 0 1 2 4

Pearson

International GCSE Mathematics

Formulae sheet – Higher Tier

Arithmetic series

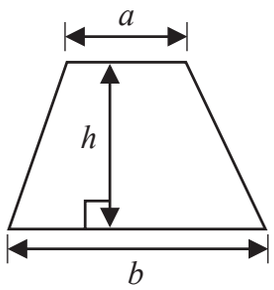
Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

The quadratic equation

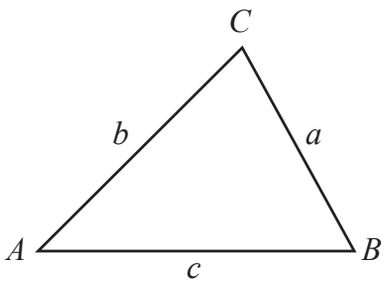
The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Area of trapezium $= \frac{1}{2}(a + b)h$



Trigonometry



In any triangle *ABC*

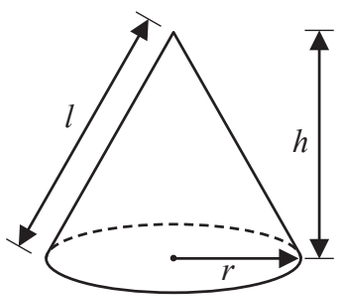
Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc\cos A$

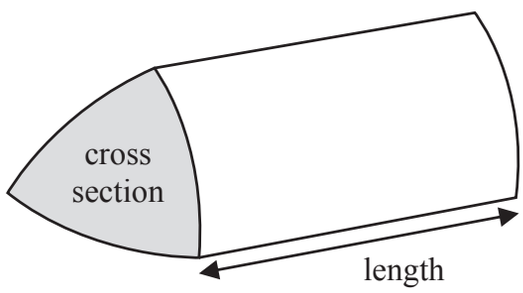
Area of triangle $= \frac{1}{2}ab\sin C$

Volume of cone $= \frac{1}{3}\pi r^2 h$

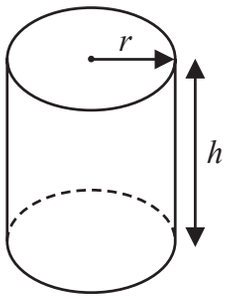
Curved surface area of cone $= \pi r l$



Volume of prism
= area of cross section × length

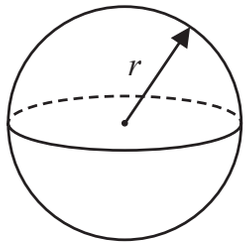


Volume of cylinder $= \pi r^2 h$
Curved surface area of cylinder $= 2\pi r h$



Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



Answer ALL TWENTY THREE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The table gives information about the number of days that 100 cars were in an airport car park.

Number of days (d)	Frequency	midpoint x	frequency × midpoint fx
$0 < d \leqslant 4$	16	2	32
$4 < d \leqslant 8$	18	6	108
$8 < d \leqslant 12$	19	10	190
$12 < d \leqslant 16$	27	14	378
$16 < d \leqslant 20$	20	18	360

(a) Write down the modal class.

highest freq = 27 class: $12 < d \leqslant 16$

(1)

(b) Work out an estimate for the mean number of days.

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{32 + 108 + 190 + 378 + 360}{100}$$

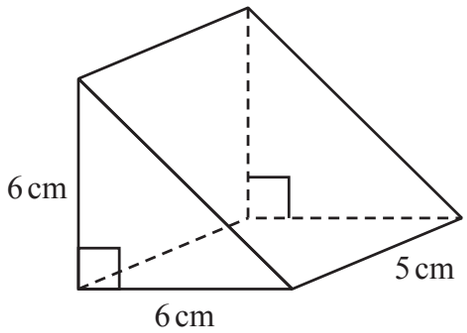
$$= \frac{1068}{100} = 10.68$$

10.68 days

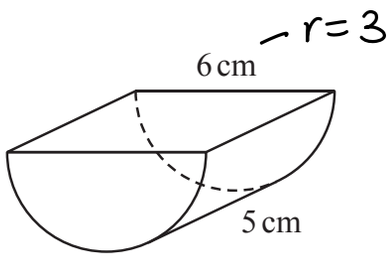
(4)

(Total for Question 1 is 5 marks)

2 The diagram shows two solid toy bricks, Brick **A** and Brick **B**.



Brick **A**



Brick **B**

Diagram **NOT** accurately drawn

Brick **A** is a triangular prism of length 5 cm.
The cross section of Brick **A** is an isosceles right-angled triangle with equal sides of length 6 cm.

Brick **B** is half a cylinder of length 5 cm.
The semicircular cross section of Brick **B** has diameter 6 cm.

The volume of Brick **A** is greater than the volume of Brick **B**.

How much greater?
Give your answer correct to 1 decimal place.

Vol = cross sectional area × height

Volume of A: $\frac{1}{2} \times 6 \times 6 \times 5 = 90\ cm^2$

Volume of B: $\frac{1}{2} \times \pi \times 3^2 \times 5$

$= \frac{45}{2}\pi\ cm^2$

Difference: $90 - \frac{45}{2}\pi = 19.314...$

= round down

19.3 cm^3

(Total for Question 2 is 4 marks)

3 Here are the first five terms of a number sequence S.

4 (−6) 10 (+6) 16 (+6) 22 28 34

(a) Find an expression, in terms of n, for the nth term of this sequence.

Difference		6
n	$\times n$	n
0	0th term	+4

$6n + 4$

(2)

The nth term of a sequence T is given by $n^2 - 3$

There are numbers that are terms in both the sequence S and the sequence T.

(b) Find one of these numbers.

Terms in both are when sequences are equal to eachother

$n^2 - 3 = 6n + 4$

$-6n \quad -4$

$n^2 - 6n - 7 = 0$

x to −7, + to −6

$(n - 7)(n + 1) = 0$

$n = 7$

can't have a negative n. $n = -1$

$n = 7$

$7^2 - 3 = 46$

$n = 7$

$6(7) + 4 = 46$

46

(2)

(Total for Question 3 is 4 marks)

4 On Saturday, Jacob walked 10 800 steps.
On Sunday, he walked 7% more steps than on Saturday.

Work out how many steps Jacob walked on Sunday.

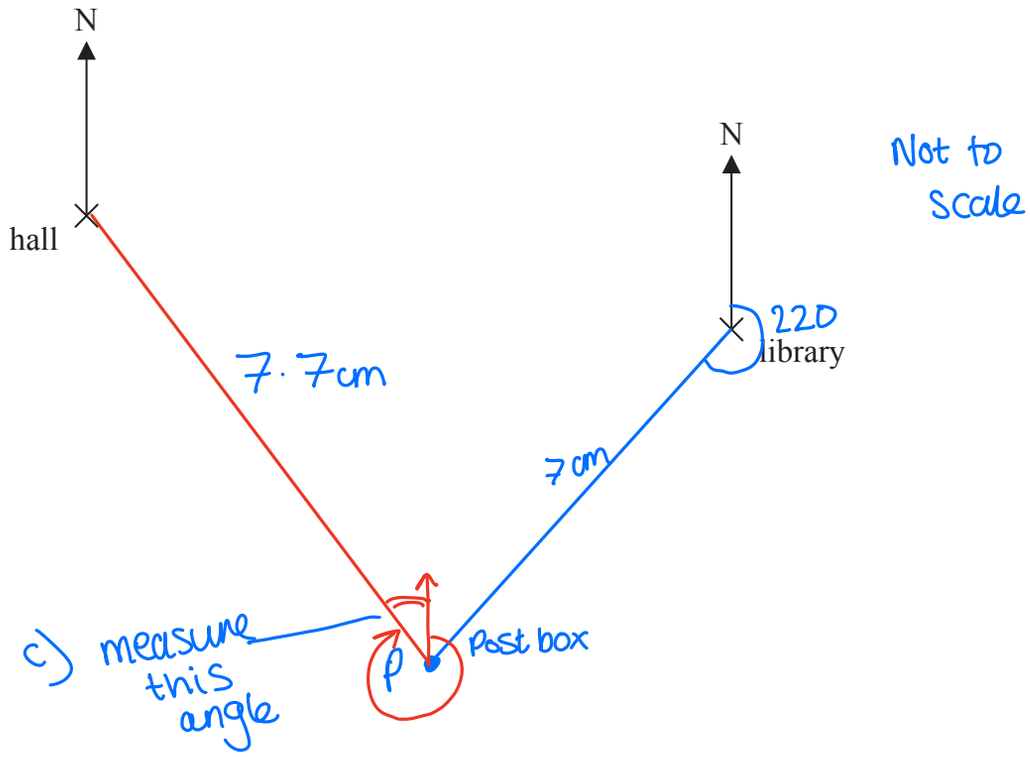
7% more: 100% + 7% = 107% = ×1.07

$10800 \times 1.07 = 11556$

11556 steps

(Total for Question 4 is 3 marks)

5 The scale drawing shows the position of a hall and the position of a library.



1cm : 20m
7cm : 140m

Scale: 1 cm represents 20 metres

A post box is 140 metres from the library on a bearing of 220°

(a) Show the position of the post box on the scale drawing.
Mark the position with a cross (×) and label it *P*.

(2)

(b) Use your scale drawing to find

(i) the real distance, in metres, of the hall from the post box,

÷7.7 (1cm : 20m
7.7cm : 154m) ×7.7

154 m

(ii) the bearing of the hall from the post box.

Angle measured : 28°

Angle around a point : 360°

360 - 28 = 332 °

(2)

(Total for Question 5 is 4 marks)

6

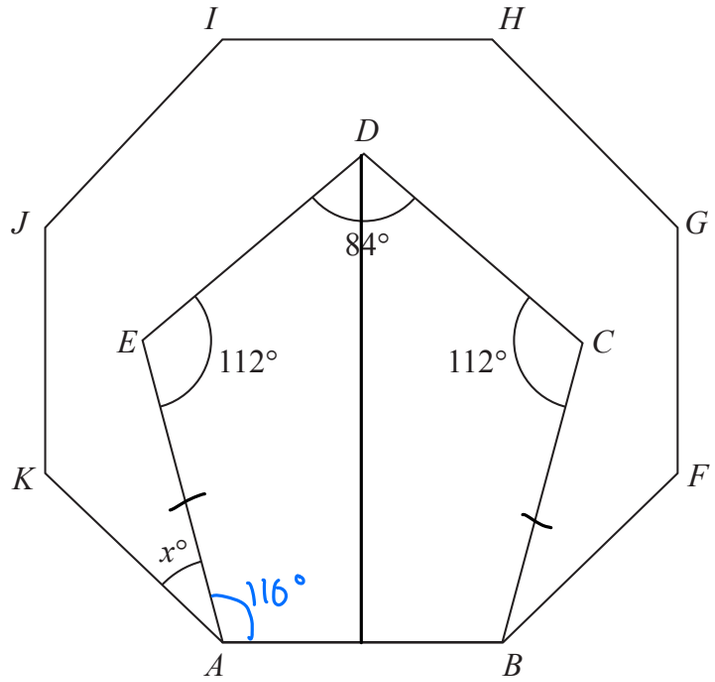


Diagram **NOT** accurately drawn

Pentagon *ABCDE* is drawn inside the regular octagon *ABFGHIJK*.
The pentagon has exactly one line of symmetry.

Work out the value of *x*.

Sum = 180(n − 2)

∠BAE : Sum of angles in pentagon : 180(5 − 2)
= 180 × 3 = 540°

112 + 112 + 84 + 2BAE = 540
−112
−112
−84
2BAE = 232
÷2
∠BAE = 116°

∠BAK : Total Angles in Octagon : 180(8 − 2)
= 1080°

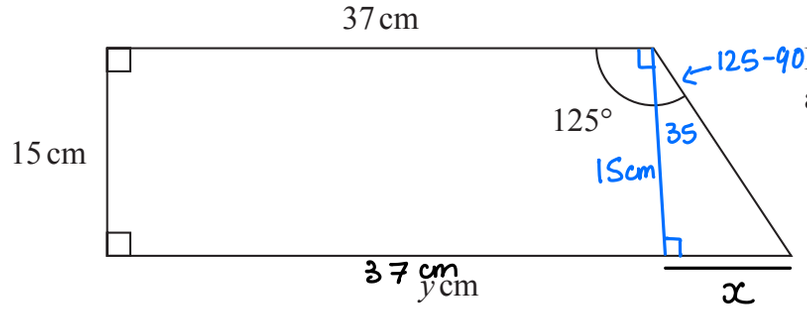
All angles are equal (regular) → 1 angle : $\frac{1080}{8}$ = 135°

x = ∠BAK − ∠BAE
= 135 − 116

$x =$ 19

(Total for Question 6 is 4 marks)

7 The diagram shows a trapezium.



Work out the value of y.
Give your answer correct to 1 decimal place.

$\tan\theta = \frac{\text{opp}}{\text{adj}}$

$\tan 35 = \frac{x}{15}$

$x = 15 \tan 35$

$y = 37 + 15\tan 35$

$= 47.503\ldots$ cm

$= 47.5$ (round down)

$y =$ 47.5

(Total for Question 7 is 4 marks)

8 (a) Simplify fully $\dfrac{15k^4m^3}{5km^2}$

$\dfrac{15k^4m^3}{5km^2} = \dfrac{3k^4m^3}{km^2} = \dfrac{3k^3m^3}{m^2} = 3k^3m$

$15 \div 5 = 3$

$k^4 \div k = k^{4-1} = k^3$

$m^3 \div m^2 = m^{3-2} = m$

$3k^3m$

(2)

(b) Solve the inequality $7 < 4x - 1 \leqslant 17$

$7 < 4x - 1 \leqslant 17$

$+1 \qquad +1$

$8 < 4x \leqslant 18$

$\div 4 \qquad \div 4$

$2 < x \leqslant 4.5$

(3)

(Total for Question 8 is 5 marks)

9 Omar invests 6000 dirham for 4 years in a savings account.
He will get 1.5% per year compound interest.

Work out the total amount of interest Omar will have received by the end of 4 years.
Give your answer correct to the nearest dirham.

After 4 years : Starting value × multiplier$^{n \text{ years}}$

Multiplier : $100\% + 1.5\% = 101.5\% = \times 1.015$

After 4 years : $6000 \times 1.015^4 = 6368.18$ (rounded 2dp)

Amount of Interest : $6368.18 - 6000$ (after 4 years, starting)

$= 368.18$

368.18 dirham

(Total for Question 9 is 3 marks)

10 (a) Simplify fully $(16x^8y^6)^{\frac{1}{2}}$ ← power of ½ = √

$= \sqrt{16} \times x^{8\times0.5} \times y^{6\times0.5}$

$= 4 \times x^4 \times y^3$

$= 4x^4y^3$

$4x^4y^3$

(2)

(b) Solve $\frac{8-2x}{3} - \frac{2x-3}{2} = 4$

Show clear algebraic working.

$\frac{2(8-2x) - 3(2x-3)}{6} = 4$ ×6

$16 - 4x - 6x + 9 = 24$

collect like terms

$-10x + 25 = 24$

−25

$-10x = -1$

÷ −10

$x = 1/10$

$x = 0.1$

(3)

(c) Make f the subject of $m = \sqrt{\frac{1}{3}ef}$

1

isolate f

square both sides

$m^2 = \frac{1}{3}ef$ ×3

$3m^2 = ef$ ÷e

$\frac{3m^2}{e} = f$

$f = \frac{3m^2}{e}$

(2)

(Total for Question 10 is 7 marks)

11 The straight line $\mathbf{L}_1$ has equation $x + 2y = 4$
The straight line $\mathbf{L}_2$ passes through the points $(-1, -7)$ and $(7, 9)$

Michael says that the lines $\mathbf{L}_1$ and $\mathbf{L}_2$ are perpendicular.

Is Michael correct?
You must show clearly how you get your answer.

gradient

$y = mx + c$

$m = \frac{y_1 - y_2}{x_1 - x_2}$

L_1 :

$x + 2y = 4$

$-x$

$2y = -x + 4$

$\div 2$

$y = \frac{-1}{2}x + 4$

$= mx + c$ → $m = \frac{-1}{2}$

L_2 :

$\frac{9 - -7}{7 - -1} = \frac{16}{8} = 2$ $\quad m = 2$

Two lines are perpendicular when the product of the gradients is -1

$2 \times -\frac{1}{2} = -1$

Yes, Michael is correct.

(Total for Question 11 is 3 marks)

12 Freddie recorded the number of runs he scored in each of 11 cricket matches.
Here are his results.

4	0	21	32	51	6	102	69	17	9	42

Find the interquartile range of his results.

In order: 0, 4, 6, 9, 17, 21, 32, 42, 51, 69, 102

(6 = LQ, 51 = UQ)

IQR = Upper Quartile (75%) − Lower Quartile (25%)

= 51 − 6 = 45

(Total for Question 12 is 3 marks)

13 Carlos, Flavia and Tazia shared £861 between themselves.

The amount of money Flavia got is 65% of the amount of money Carlos got.
The amount of money Tazia got is 22% **more** than the amount of money Carlos got.

Work out how much money Carlos got.

Carlos get £x.

Flavia gets 65% of x = $0.65x$

Tazia gets: 22% more:
100% + 22% = 122% of x = $1.22x$

$x + 0.65x + 1.22x = 861$

$2.87x = 861$

$\div 2.87$

$x = 300$

£ 300

(Total for Question 13 is 3 marks)

14 (a) Given that $a = 3^x$ and $b = 3^y$

express in terms of a or b or a and b,

(i) 3^{2x}

$= 3^{2 \times x} = (3^x)^2 = a^2$

(ii) 3^{x+4y}

$= 3^x \times 3^{4y}$

$= 3^x \times (3^y)^4 = ab^4$

(iii) 3^{y-1}

$= 3^y \div 3^1$

$= b \div 3$

$\frac{1}{3}b$

(3)

$a = 3^x$ and $b = 3^y$

$ab = 2187$

$a^2b = 177\,147$

(b) Work out the value of x and the value of y.
Show your working clearly.

$a^2b \div ab = a$

$177147 \div 2187 = 81$

$3^x = 81$

$3^4 = 81$ $\quad x = 4$

$a \times b = 2187$

$81 \times 3^y = 2187$

$\div 81$

$3^y = 27$

$3 \times 3 \times 3 = 27$

$y = 3$

$x =$ 4

$y =$ 3

(3)

(Total for Question 14 is 6 marks)

15 Barney has a biased coin.

When the coin is thrown once, the probability that the coin will land heads is 0.3

Barney throws the coin 4 times.

(a) Work out the probability that the coin will land heads exactly 3 times.

Heads 3 times: $0.3^3 \times 0.7 = 0.0189$

4 different ways of exactly 3 : 0.0189×4

$= 0.0756$

(3)

(b) Work out the probability that the coin will land heads at least once.

$= 1 - P(\text{no head})$

$= 1 - 0.7^4$

$= 0.7599$

(2)

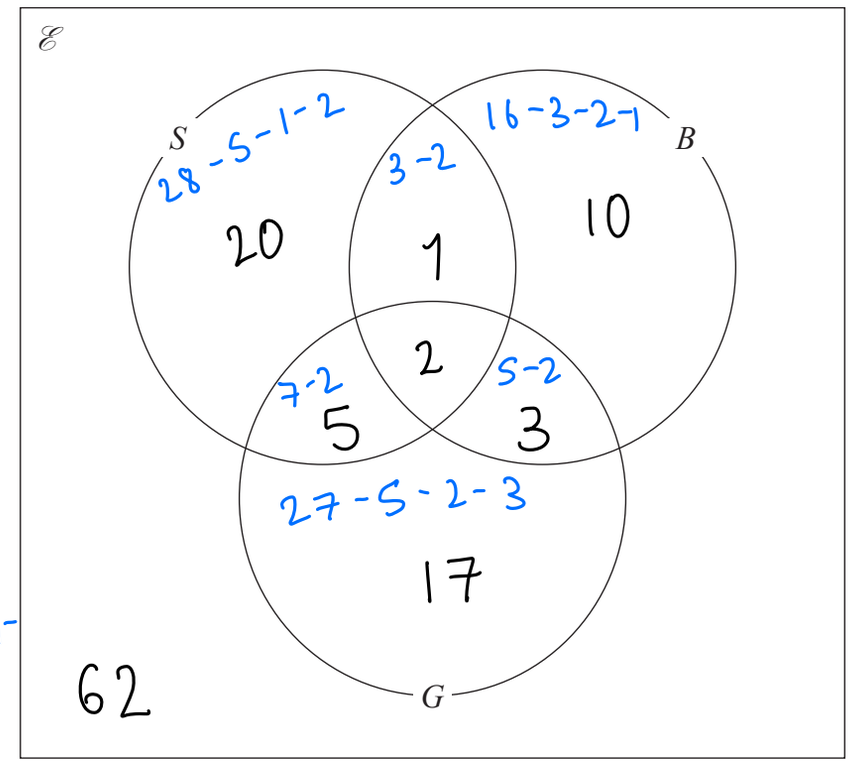
(Total for Question 15 is 5 marks)

16 120 people who visited a sports centre were asked if they went swimming (S), played basketball (B) or used the gym (G).

Their answers showed that

28 people went swimming
16 people played basketball
27 people used the gym
3 people went swimming and played basketball
5 people played basketball and used the gym
7 people went swimming and used the gym
2 people went swimming, played basketball and used the gym

(a) Using this information, complete the Venn diagram to show the number of people in each region of the Venn diagram.



120-20-10-17-5-2-1-3=

(3)

One of the people who went swimming is chosen at random.

(b) Find the probability that this person also played basketball.

28 went swimming

3 also play basketball

$\frac{3}{28}$

(1)

(Total for Question 16 is 4 marks)

17 $P = ef$

Upper and lower bounds

e = 4.8 correct to 2 significant figures.
f = 0.26 correct to 2 significant figures.

(a) Work out the lower bound for the value of P.
Show your working clearly.
Give your answer correct to 3 significant figures.

$4.75 \leqslant e < 4.85$

$0.255 \leqslant f < 0.265$

LB of P = LB × LB

$= 0.255 \times 4.75$

$= 1.21125$

$= 1.21$ (3sf)

(2)

$Q = \dfrac{t}{w}$

t = 2.73 correct to 3 significant figures.
w = 0.04 correct to 1 significant figure.

UB = UB / LB

(b) Work out the upper bound for the value of Q.
Show your working clearly.
Give your answer correct to 2 significant figures.

$2.725 \leqslant t < 2.735$

$0.035 \leqslant w < 0.045$

$= \dfrac{2.735}{0.035} = 78.142\ldots$

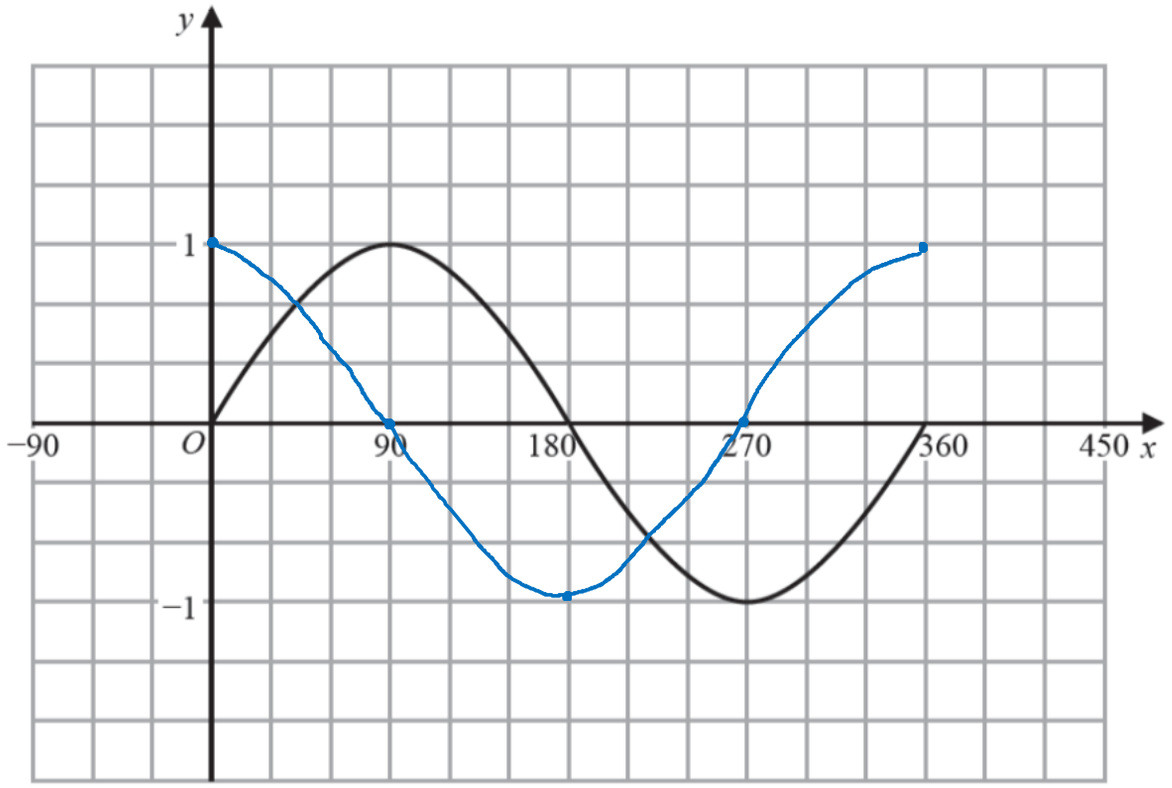
= round down

78 (2sf)

(2)

(Total for Question 17 is 4 marks)

18 Here is the graph of $y = \sin x°$ for $0 \leqslant x \leqslant 360$



(a) On the grid above, sketch the graph of $y = \sin(x + 90)°$ for $0 \leqslant x \leqslant 360$

move graph to the left by 90°

(2)

In $0 \leqslant x \leqslant 360$, the graph of $y = \sin\left(\frac{x}{2}\right)° + 3$ has a maximum at the point A.

move up 3

÷ x values by ½ (×2)

(b) Write down the coordinates of A.

Max point before: (90, 1)

After transformation (90×2, 1+3) (180 , 4)

(2)

(Total for Question 18 is 4 marks)

19 *ABCD* is a quadrilateral.

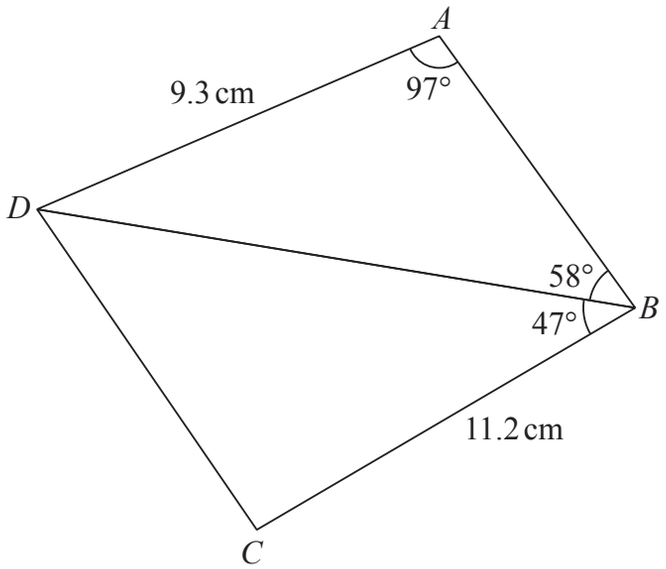


Diagram **NOT** accurately drawn

$\frac{a}{\sin A} = \frac{b}{\sin B}$

Find the area of quadrilateral *ABCD*.
Give your answer correct to 3 significant figures.

Area $= \frac{1}{2} ab \sin C$

$\frac{BD}{\sin 97} = \frac{9.3}{\sin 58}$

$BD = \frac{9.3 \sin 97}{\sin 58} = 10.8846...$

$\angle BDA = 180 - 97 - 58 = 25°$

Area of $\triangle ABD$: $\frac{1}{2} \times 9.3 \times 10.8846... \times \sin 25$

$= 21.39... \text{ cm}^2$

Area of $\triangle BCD$: $\frac{1}{2} \times 11.2 \times 10.8846... \times \sin 47$

$= 44.57... \text{ cm}^2$

Area of ABCD $= 21.39.. + 44.57..$

$= 65.96$ ← round up

66.0 cm²

(Total for Question 19 is 5 marks)

20 (a) Write $3x^2 - 12x + 7$ in the form $a(x + b)^2 + c$

Take out factor 3 → $3x^2 - 12x + 7$

$3(x^2 - 4x) + 7$

complete the square

$-4 \div 2 = -2$

$3[(x - 2)^2 - 4] + 7$

expand out [ ] brackets

$3(x - 2)^2 - 12 + 7$

simplify

$3(x - 2)^2 - 5$

(3)

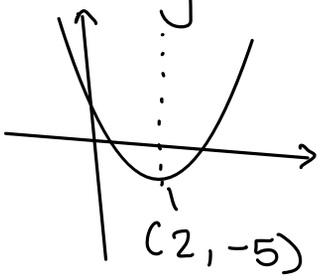
The line **L** is the line of symmetry of the curve with equation $y = 3x^2 - 12x + 7$

(b) Using your answer to part (a) or otherwise, write down an equation of **L**.

$a(x + p)^2 + q$

Turning point $= (-p, q)$

$= (2, -5)$



symmetry $x = 2$

(1)

(Total for Question 20 is 4 marks)

21 The curve with equation $y = (10x - 3)(x + 1)$ ① and the line with equation $y = 6x$ ② intersect at the points A and B.

Find the coordinates of the midpoint of AB.
Show your working clearly.

① $y = (10x - 3)(x + 1)$

$y = 10x^2 + 10x - 3x - 3$

$y = 10x^2 + 7x - 3$

② $y - 6x = 0 \quad \therefore y = 6x$

Equate both:

$10x^2 + 7x - 3 = 6x$

$10x^2 + x - 3 = 0$

x to $-3 \times 10 = -30$
+ to 1
$-5, 6$

$10x^2 - 5x \mid +6x - 3 = 0$

$5x(2x - 1) \mid 3(2x - 1)$

split and factorise each side

$(5x + 3)(2x - 1) = 0$

$x = -3/5$ $\quad x = 1/2$

$y = 6x$

$\times 6 \qquad \times 6$

$y = -18/5 \qquad x = 3$

A: $(-0.6, -3.6)$ $\quad$ B: $(0.5, 3)$

Midpoint: $\left(\frac{0.5 + (-0.6)}{2}, \frac{3 + (-3.6)}{2}\right)$

$(-0.05, -0.3)$

(Total for Question 21 is 6 marks)

22

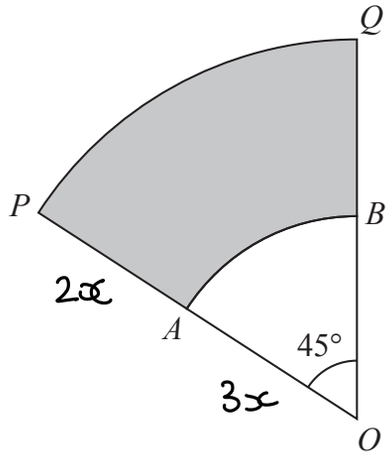


Diagram **NOT** accurately drawn

OPQ is a sector of a circle, centre *O*
OAB is a sector of a circle, centre *O*

A is the point on *OP* such that $OA : AP = 3 : 2$
B is the point on *OQ* such that $OB : BQ = 3 : 2$
Angle $POQ = 45°$

The area of the shaded region is $\frac{81}{2}\pi$ cm^2

Work out the perimeter of the shaded region.
Give your answer in terms of π.

Area of OPQ $= \pi \times (5x)^2 \times \frac{45}{360}$ ↙ $\frac{1}{8}$

$= \frac{25x^2\pi}{8}$

Area of OAB $= \pi \times (3x)^2 \times \frac{1}{8}$

$= \frac{9x^2\pi}{8}$

Area of shaded : $\frac{25x^2\pi}{8} - \frac{9x^2}{8} = \frac{16x^2\pi}{8}$

$2x^2\pi = \frac{81}{2}\pi$

× 2

$4x^2 = 81$

$\div 4$

$x^2 = \frac{81}{4}$

$\sqrt{\ }$

$x = \frac{9}{2}$ – ignore negative value as length can't be negative

Circumference PQ : $2\pi \times (5 \times \frac{9}{2}) \times \frac{45}{360}$ – $\frac{1}{8}$

$= \frac{45}{8}\pi$

Circumference AB : $2\pi \times (3 \times \frac{9}{2}) \times \frac{45}{360}$

$= \frac{27}{8}\pi$

AP and BQ = $2 \times \frac{9}{2} = 9$

Perimeter : $9 + \frac{27}{8}\pi + \frac{45}{8}\pi + 9 = 18 + 9\pi$ cm

(Total for Question 22 is 6 marks)

Turn over for Question 23

23 The 10th term of an arithmetic series, S, is 66
The sum of the first 20 terms of S is 1290

Find the 5th term of S.
Show your working clearly.

a: 1st term
d: difference

$a + 9d = 66$ ×20

Sum of seris $= \frac{n}{2}(2a + (n-1)d)$

$1290 = \frac{20}{2}(2a + (20-1)d)$

$1290 = 10(2a + 19d)$

$1290 = 20a + 190d$

$20a + 190d = 1290$
$20a + 180d = 1320$ −
$10d = -30$
$d = -3$

$a + 9d = 66$
$a = 66 - (9 \times -3)$
$= 66 + 27$
$a = 93$

5th term:
$a + 4d$
$93 + 4(-3)$
$93 - 12$
$= 81$

(Total for Question 23 is 4 marks)

TOTAL FOR PAPER IS 100 MARKS